FINITE DIFFERENCE PROGRAM FOR SOIL PROPERTIES BY THE METHODE OF CHARACTERISTICS

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خلاصة البحث

يستخدم البحث الحالي تقنية الفروق المحددة لحل معادلات التوازن والتوافق والإجهاد-الانفعال لمنحنيات الفشل التي تحصل للترب بجميع أنواعها وتحت أي نوع من الأحمال الساكنة وباسلوب معادلات الخصائص (Equation of characteristics), إذ تم عمل برنامج حاسوبي لحساب شبكة الفشل في الترب المتعرضة للأحمال الساكنة بعد تقسيمها إلى ثلاثة مناطق وباستخدام لغة فورتران (77). وقد تم التحقق من كفاءة التقنية والبرنامج معاً بحل عدة حالات ومن ثم حل ثلاثة مشاكل أساسية في التربة وهي الأحمال، ثبوتية المنحدرات، والجدران الساندة.

ABSTRACT

This work deals with the limit equilibrium technique, which is more approximate than the limit analysis. Besides it deals with the mechanism of falling surfaces and slip surfaces occurring in soils of different cases. The procedure followed in this work is to put the equilibrium equations by taking element from soil under general condition. Solving these two partial differential equations by the method of characteristics in order to get two hyperbolic ordinary equations, then solving these equations by the finite difference method using computer algorithm programmed using *FORTRAN 77* language in order to give the complete solution of the problem in concern. The proposed solution will provide the advantages to deal with the conditions of soil at any point (stresses, failure slopes) under different effects such as surcharge, water effect, retaining walls effect… etc. This means any failure surface of any type of distribution of load or stress in any kind of soil can be solved using the proposed technique. Finally many checks are made to insure the validity of the proposed technique.

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Introduction and theoretical background: The soil must be capable of carrying the loads from any type of structure founded upon it with acceptance settlements but without causing any type of failure.

So, it is necessary to investigate phenomenon, stress and deflection (base shear resistance and settlement) for any structure. In many cases settlement criteria will control the allowable bearing capacity; however, there are also a number of cases where base shear dictates the recommended bearing capacity, e.g. the allowable bearing capacity for footings on unsaturated cohesive soil which is often based on the unconfined compression strength, which is turn a simplified version of the bearing capacity equations. Structure founded on soft soils such as liquid storage tanks and mats may be more susceptible to a base shear failure than to settlements, particularly if the structure has such a loading that it settles uniformly and where a large amount of settlement can be tolerated.

In the present problem the shear strength phenomena is put in a certain formulas that enable the researchers to evaluate and draw the magnitude and flow of the failure stresses in any type of granular soil under any type of construction or load and due to surface inclination.

The proposed procedure deals with more exact analysis depending on **FDM "Finite Deference Method"** which gives as approximate as the actual shape of failure lines (slip lines).

Figure (1) expresses the previous introduction by the simple way of analysis, (A): Case of active resistance, (B): Case of passive resistance. A more developed procedure can be adopted by dividing the failure mesh into (3) zones (active zone, intermediate transmission zone, and Passive zone -see fig.2-).

A more developed procedure is adopted which represents the general case of failure zones and it is illustrated in figure (3). Generally there are two essential techniques dealing with stress field computations:

1. **Limit equilibrium approach**

The limit equilibrium method has been used to obtain approximate solution for stability problems in soil mechanics. The method can probably best be described as an approximate approach to construction of the slip line field and generally entails an assumed failure surface of various simple shapes – plane, circular or log spiral. This approach is basically depending on three conditions stress equilibrium equation, stress boundary condition, and yield criterion.

In this method it is necessary to make sufficient assumptions regarding the stress distribution along the failure surface, such that an

over all equation of equilibrium, in terms of stress resultants, may be written for a given problem. So, various cases and problems can be solved by simple static in admissible stress field of the problem under consideration. The equilibrium equations are:

$$\partial \frac{\sigma_x}{\partial_x} + \partial \frac{\tau_{xy}}{\partial_y} = \gamma \sin \alpha'$$

$$\partial \frac{\tau_{xy}}{\partial_x} + \partial \frac{\sigma_y}{\partial_y} = \gamma \cos \alpha'$$

2. Limit analysis approach

Generally any computations needed in the mechanism of deformable solids in the elastic range of strain based on three conditions stress equilibrium equation, strain- stress relationship, and compatibility equations relating strained displacement.

The soil behavior is an elastic-plastic behavior, which needs three-stages of development initial plastic response, intermediate plastic flow, unrestricted plastic flow.

Limit analysis of the method, which enables definite statement to be made about the collapse load without carrying out the step, by elastic-plastic analysis. This technique can further be divided by:

Lower bound:

Lower bound is related to what is called statically admissible stress field which accurse when distribution of load on soil satisfied the equilibrium equation, stress boundary condition, no way of violating the yield criterion or the actual collapse loads (existing stress < the collapse load).

Then a statically admissible stress distribution is occurred, which means a uncontained plastic flow will not occur at this load stage, e.g. the lower bound take no consideration of kinematics behavior of soil.

Upper bound

The load (p) is coming from equating the *External rate of work* to the *Internal rate of deformation mode in the velocity field.* The load (p) must be equal or larger than the collapse load, and the velocity field must satisfy the velocity boundary condition, and the strain velocity compatibility equation.

Then a kinematically admissible field is applied and the dissipation of energy in such field can be computed from what is called flow rule, which is the plastic strain rate, divided by the current stress. Essentially flow rule is the transformation of the Hook's law to a formula of (σ_E) in a plastic zone, so the total deformation will be :

$$\xi_T = \xi_E + \xi_P$$

The upper bound theory says: if a kinematically admissible field exists then plastic flow will be valid, e.g. this technique considers only velocity of failure mode and dissipation of energy, but no need for satisfying the stress equilibrium equation.

By using the Lower – Upper bounds theorems, the exact load can be computed.

The Method of Characteristics

The method of characteristics can be simply defined as a path of transmission or propagation of energy. In other word, the change of variables of different kind depends on the problem through out a path or a line called a characteristic line.

To make the view of characteristics more sensible, a path of a vehicle passing through a road in a certain time with the effect of its wheels on the road can be taken as an example for the characteristics lines. The lines of tires of that vehicle along the road and the transmission of their loads on the geological mass with the disturbance caused by theses loads are called a ***characteristics lines***, and the relationship that collect load, disturbance and time can be formed in a ***characteristic function***.

If a propagation of wave and displacement with time a long a one-dimensional steel rod with medium length as shown in fig. (4-a) the paths of propagation of displacement (u) in the plane (xt) where (x) is the direction in which the rod length is measured and (t) is the time where its direction represent the change of displacement along the length of the rod. If two-dimensional propagation of the characteristics between (x & t) -as shown in figure (4-b)- is to be studied then the propagation can take place in two directions, left to right and right to left. The first propagation (left to right) is denoted (C+) characteristics and the other propagation is denoted (C-) characteristics.

Solution of the problem involves finding values of (u) over the (xt) plane is to find the value of the characteristics. The over two-direction propagation between (u & t) also transmit along the same characteristic of the previous two-dimensional paths of u along (xt) plane, e.g. the characteristics of the velocity (v) is:

$v = \frac{du}{dt}$ = Velocity of propagation is the same characteristics of the displacement (u).

Generally there are number of factors that influence the paths of propagation of variables in any problem deals with characteristics such as the initial conditions, laws governing the problem (often expressed by set of differential equations), and the physical properties of the material.

Method of characteristics deals with many kinds of problems such as the deflection in strings struts and beams, pressure and sound waves, long period waves, and the stress waves.

Mathematical definition of characteristic solution:

Simply characteristics solution is a set of partial deferential equations that converted in ordinary differential equations by using the advantages of auxiliary values, and then solved numerically by finite differences method.

The characteristic solution can solve different kinds of differential equation sets. This solution can solves either Partial differential equation, or ordinary differential equation (up to n^{th} order).

In other words, characteristic solution is the general solution (which would turn to be particular complete solution when the boundary condition are valid) for linear homogeneous or non-homogeneous differential equations of a hyperbolic type.

An **Eigen value** or characteristic or latent root of n-rowed (real or complex) square matrix A ($A = a_{j,k}$) is defined as a number (λ) such that the vector equation ($Ax = \lambda.x$) has a non-trivial solution when $x \neq 0$, which is called **Eigen vector** or characteristic value of (A) corresponding to the value of (λ). The **Eign values** of (A) are the roots of the characteristic equation:

$$(D(\lambda) = \det.(A - \lambda I) = 0)$$

Where (I) is the n- rowed unit matrix and $D(\lambda)$ is the characteristic determinant. By developing $D(\lambda)$ one can obtain a polynomial of n^{th} degree in (λ) which called characteristic polynomial corresponding to (A). The value of ($a_{j,k}$) in the matrix- determinate and polynomial may be variable or constant.

By substituting α & β in the previous equations, one can have the following:

$$\frac{\partial \gamma}{\partial \alpha} = \frac{\partial x}{\partial \alpha}\tan(\psi - \varepsilon) \qquad \frac{\partial y}{\partial \beta} = \frac{\partial x}{\partial \beta}\tan(\psi + \varepsilon)$$

$$\frac{\partial \sigma}{\partial \beta} + 2\sigma \tan\varphi \frac{\partial \psi}{\partial \beta} = \frac{\gamma}{\cos\varphi}\left[\sin(\bar{\alpha} + \varphi)\frac{\partial x}{\partial \beta} + \cos(\alpha + \varphi)\frac{\partial y}{\partial \beta}\right]$$

$$\frac{\partial \sigma}{\partial \alpha} - 2\,\sigma \tan\varphi \frac{\partial \psi}{\partial \alpha} = \frac{\gamma}{\cos\varphi}\left[\sin(\bar{\alpha} - \varphi)\frac{\partial x}{\partial \alpha} + \cos(\bar{\alpha} - \varphi\frac{\partial y}{\partial \alpha})\right]$$

All terms of these equations shall be defined later. By the usage of the previous definitions one shall have the characteristic determinate of these non-homogeneous differential equations. In the following formula:

$$D = \frac{\partial (x, y)}{\partial (\alpha, \beta)} \neq 0$$

By assuming very convenient symbols: P = equivalent stress, L = characteristic length along the loaded area and by substituting the non-dimensional variables in the main equations and getting use of some relations like:

$$\left.\begin{matrix}\sigma x \\ \sigma y\end{matrix}\right\} = \sigma(1 \pm \sin\varphi \cos 2\psi) - H$$

$$\tau xy = \sigma \sin\varphi \sin 2\psi$$

In addition to some relations coming from geometry, then rearranging, one can have the following form:

$$\left[\left(\frac{\delta}{\delta x}, \frac{\partial}{\partial y}\right) [\sigma, \psi] \right]$$

Three cases are to be performed:

(1) If d_1, $d_2 \neq 0 \rightarrow$ values of dx ,dy are uniquely valid.

(2) If d_1, d_2 vanish simultaneously with (n_1, n_2) $\rightarrow$ dx, dy are not uniquely valid and then the equation Of line y = f (x) in the plane (xy) $\rightarrow$ characteristic line.

(3) If (n_1, n_2) $\neq$ 0 & (d_1,d_2) & (d_1, d_2) vanish with them simultaneously $\rightarrow$ (dy, dx) $\rightarrow$ $\infty \rightarrow$ y = f(x) coincide with the line of discontinuity.

By taking case (2) & solving (n_1,n_2) = 0 , (d_1,d_2) =0 simultaneously , then the characteristic differential equation are to be established.

$$dy = dx \tan(\omega \pm \varepsilon) \qquad \text{(c1)}$$

$$d\sigma \pm 2\sigma \tan\varphi \, d\psi = \frac{\gamma}{\cos\varphi}\left[\sin(\bar{\alpha} \pm \varphi)\, dx + \cos(\bar{\alpha} + \bar{\varphi}) dy\right] \qquad \text{(c2)}$$

(c_1 & c_2) are the ordinary differential equation of characteristic. The equations of (-) sign called first family of characteristic, and (+) sign called second family of characteristic. Both represent two different families of the characteristic of hyperbolic type. The characteristic line that inclined from x by an angle of ($\psi \pm \varepsilon$) since slip lines are at ($\psi \pm \varepsilon$) angle with x, so slip lines coincident with the characteristic lines which they are (α, β) lines. By substituting (α, β) in the equation of characteristic one will have the equation this gives and describes the limiting equilibrium in (α, β) lines only, and the values of σ, ψ, x & y along these lines. So by solving the equation of the characteristic by analytic methods or numerically one shall have the families of lines of failure that correspond the mish of the failure zone.

The present work description:

Solution of the characteristic equation by F.D.M.: If one considers $\alpha` = 0$, $dy = dx \tan(\psi \pm \varepsilon)$, $d\sigma = 2\sigma \tan\varphi\, d\psi = \gamma(\tan\varphi\, dx \pm dy)$, then the numerical solution will base on dimensional variables.

$$\bar{x} = \frac{x}{l} \qquad \bar{y} = \frac{y}{l} \qquad \bar{\sigma} = \frac{\sigma}{\gamma l}$$

l : characteristic length.
P: characteristic stress.

In order to find (x,y,σ,ψ) of the unknown point in the mesh of failure zone, the following points must be adopted:

(1) First assumption:
$x = x1 + x2 - x3 \qquad \sigma = \sigma1 + \sigma2 - \sigma3$
$y = y1 + y2 - y3 \qquad \psi = \psi1 + \psi2 + \psi3$
If values of point (3) are not available, then: [[$x=1/2\,(x1+x2)$, $y=1/2\,(y1+y2)$, $\sigma=1/2\,(\sigma1+\sigma2)$, and $\psi=1/2\,(\psi1+\psi2)$]].

(2) From first assumption x,y coming from above and substituting their values (x,y) in equation (c1`) b & (c2`) one shall have the values of $\sigma1$ & ψ.

(3) Solving equation (c1`)a & (c2`)a by using off values of $[\psi, \sigma1]$ coming from assumption (1)one will have values of x and y.

(4) By checking the values x,y,σ,ψ coming from (2-3) with values of x,y,σ,ψ from assumption (1).
If not good → By assuming another values of (x,y,σ,ψ) and repeat (2,3,4).

The above technique can be arranged in a computer program in order to find the true values of (x,y,σ,ψ).

Details for general problems

Basically the problem in concern can be solved by dividing the mesh (Failure zone) into three zone (minor problems):

1. **Cauchy problem (zone « 1 »)**
2. **Goursat problem (zone « 2 »)**
3. **Mixed problem (zone « 3 »)**

Generally zone "1" has the initial conditions of (x,y,σ,ψ) of the free surface (boundaries) of the axis on which the load "p" or "q" have been exerted. So, the values (x,y,σ,ψ) are very important to solve the first

problem (Cauchy). The equations that give the values of previous terms could be found elsewhere[5] . The general equations are:

A. $\psi = (1-\kappa)\frac{\pi}{4} + \frac{1}{2}(\kappa\kappa - \delta) + m\pi \qquad \sigma = p\frac{\sin\Delta}{\sin(\Delta - \kappa\delta)}$

B. when $\delta = 0 \Rightarrow \quad \psi = (1-\kappa)\frac{\pi}{4} + m\pi \qquad \sigma = \frac{p}{1-\kappa\sin\varphi}$

C. The most general case (when there is an inclination of the ground & the load).

$\Rightarrow \psi = \frac{\pi}{4}(1-\kappa) + \frac{1}{2}(\kappa\kappa - (\delta + \mu * B) + m\pi$

$\sigma = p\,\sin\Delta$

$\sin(\Delta - \kappa(\mu\beta + \delta))$

The previous equations are general limiting equilibrium equation for (σ) and (ψ) in the free surface (negative or positive axis) with regard of: - ($\kappa = \pm 1$), ($\mu = \pm 1$), ($m = \pm 1,0$), $\sin\Delta = \sin\delta / \sin\varphi$. ($\delta$: angle of inclination of load with vertical) (B: angle of the inclination of load with horizontal). The sign of (κ, μ) depend on the case of unbalancing and the direction of x-axis or any inclined axis of the free surface one can calculate the initial condition.

Solution of all zones

After knowing the initial values of three zones one can now solve the three zones by the main equation. of limiting equilibrium: -

By iteration the first assumption certain value for (x,y,σ,ψ) can be predicted, and by using the following ***5 points*** below the program can be run successfully:

(1.) Initial condition, **(2.)** First assumption, **(3.)** The previous (4 equations), **(4.)** Flow chart of program, and finally **(5.)** Data of the case.

Case Studies

Case 1:*Normal restraining pressure on foundations

A model which was first suggested by *khristianovich,* the most typical example of a problem on the limiting equilibrium of a granular medium is the determination of the carrying capacity of a foundation under the action of normal or inclined pressure.

Obviously, there are two solutions to this problem, one when the load is less than p, and the other when it is greater. The movement resulting from the loss of equilibrium takes place in opposite directions for the two cases. For detailed examination of the stability of a foundation bounded by the (x-axis) supposing that an equivalent normal pressure p= p (x) is applied along the positive semi-axis of x. The equivalent normal pressure

(q), which must be applied along the negative semi-axis of x, is determined so that the foundation remains in limiting equilibrium without heaving or settlement. It's assumed that the foundation p (x) is continuous and that it has a continuous derivative.

The normal and tangential components of the equivalent pressure along the positive and negative semi-axis of (x) can be expressed in the following ways:

$$\sigma y + H = p \quad , \tau xy = 0 \qquad \text{and} \qquad \sigma y + H = q \quad , \quad \tau xy = 0$$

This problem has two solutions: one determines the restraining pressure, i.e. the pressure at which there will just be no heaving along the negative semi-axis of (x). The second solution determines the failure pressure, i.e. the pressure at which there will just be no settlement along the negative semi-axis of x: the restraining pressure is usually several times less than the corresponding failure pressure.

The first step is finding the restraining pressure on a foundation assuming that loss of a limiting equilibrium leads to a settlement of the foundation along the positive semi-axis of (x) (a lowering of the left hand pan of the balance), and a heaving along the negative semi-axis of x (a

$$\sigma = \frac{p(x)}{1+\sin\varphi} \qquad , \quad \varphi = \frac{\pi}{2} \quad (\text{ from } \qquad \sigma = \frac{p}{1-\kappa \sin\varphi})$$

raising of the right-hand pan). Therefore, in the zone adjacent to the positive semi-axis of (x), a minimal stress state will exist and in the zone adjacent to the negative semi-axis of (x), a maximal stress state will exist also. Thus along the positive semi-axis of x, with ($\kappa = -1$):

While along the negative semi-axis of x, according to the previous equation. ($\sigma = p / (1-\kappa \sin\varphi)$) with ($\kappa = +1$), (q) will be equal to ($\sigma (1-\sin\varphi)$, $\varphi = \pi$). The input, output and sketch of this case are shown in figure (6).

Case 2: Stability of Slopes

Considering the limiting equilibrium of a medium bounded by the x-axis, which is inclined to the horizontal at an angle α and along which there acts a uniformly distributed equivalent pressure p at a constant angle.

The normal and tangential components of the equivalent pressure along the x-axis can be expressed, as usual, in the form:

$$\sigma y + H = p\cos\delta \qquad , \qquad \tau xy = p\sin\delta \qquad , \text{ where } |\delta| \le 0$$

The stress state set up in the body is independent of x and can be described by the differential equations of equilibrium. Thus the

equivalent stress acting on inclined straight lines parallel to the x-axis is given by the equations.

$$\sigma = py\,\frac{\sin \Delta y}{\sin(\Delta y - \kappa\,\delta y)} \quad , \quad \varphi = (1-\kappa)\,\frac{\pi}{4} + \frac{1}{2}(\kappa\,\Delta y - \delta y) + m\pi$$

and can be expressed in terms of δy or φ as follows: -

$$y = -\frac{p}{\gamma}\,\frac{\sin(\delta - \delta y)}{\sin(\alpha - \delta\, y)} = -\frac{p}{\gamma}\,\frac{\sin\delta - \sin\rho\,\sin(2\varphi + \delta)}{\sin\alpha - \sin\rho\,\sin(2\,\varphi + \alpha)}$$

where by definition: –

$$\sin \Delta y = \frac{\sin \delta y}{\sin \rho} \quad , \quad |\Delta y| \le \frac{\pi}{2}$$

The two values of (κ, -1 and +1) appearing in this solution correspond to the two stress states – the minimal and maximal. So, a straight line on which $\tau xy = 0$ can be drawn; it corresponds to $\delta = 0$, so If $\alpha \le \rho$, limiting equilibrium is possible over the whole half-plane $0 \le y \le \infty$, since the inequality $|\delta y| \le \rho$ is satisfied automatically if, however, $\alpha > \rho$, limiting equilibrium is possible only within the strip $0 \le y \le y^*$ in which the inequality $|\delta y| \le \rho$ is valid.

One can find without difficulty the slip-lines for the stress state under consideration. In order to do so, one must express y in terms of ψ in the differential equations:

$$\frac{dy}{dx} = \tan(\varphi \pm \varepsilon)$$

$$y - y_0 = b\sin(\alpha - \varphi)\,\frac{\sin 2\varphi}{\sin\alpha - \sin\rho\,\sin(2\varphi + \alpha)}$$

$$y = y_0 = -\frac{p}{\gamma}\,\frac{\sin\delta}{\sin\alpha}$$

The slip-lines, which are applicable to the minimal stress state for angle of internal friction $\varphi = 30^o$ and for $\alpha = 45^o$.

Note that the line $y = y^*$ parallel to the boundary serves as an envelope of the first family of slip-lines and consequently is a line of discontinuity. Indeed the slop of the family of slip-lines:

$$\frac{dy}{dx} = \tan(\varphi \pm \varepsilon)$$

On this line vanishes, since $\varphi = \varepsilon$.

Also note that the particular case when the equivalent pressure p on the x-axis acts vertically or when ($\delta = \alpha$). Obviously:

$$\sigma y + H = (p + \gamma y)\cos\alpha \quad , \quad \tau xy = (p + \gamma y)\sin\alpha$$

The input, output and sketch of this case are shown in figure (7).

Case 3: Pressure of a fill on retaining walls

A: Active pressure resistance of a fill on retaining walls

The determination of the pressure of a fill on a retaining wall when frictional forces act on the back of the walls is one of the classical problems the limiting equilibrium of a granular medium. The movement of which is resisted by considerable friction in the guides and bearing, can quite conveniently illustrate this problem.

It shall be attempted to find the pressure in the rear face of a retaining wall inclined at an angle (β) from a fill bounded by the positive semi-axis of (x). One can assume that an equivalent normal pressure (p = p (x)) is applied along this semi-axis, and one shall try to find the equivalent pressure q acting on the rear face at an angle (δ) with the normal.

For sufficient large values of the angle (β) the stress distribution when the fill is in a state of limiting equilibrium can be continuous every where except at the point (O), provided the equality:

$$|\delta| = w \leq \varphi$$

holds on the rear face of the wall. The normal and tangential components of the equivalent pressure acting on the rear face can be expressed as follows:

$$\sigma n + H = q \cos\delta \quad , \quad \tau_n = q \sin\delta \quad , \text{ where } |\delta| \leq \varphi$$

This problem has two solutions: are gives the active pressure- the actual force per unit area exerted by the fill, and the other gives the passive pressure minus the reactive force per unit area which the fill is capable of exerting on a moving retaining wall; the active pressure is usually several lines less than the corresponding passive pressure.

First one can determine the active pressure of a fill on a resists the downward slip of the fill, so that ($\delta = \omega \geq 0$). In the zone adjacent to the positive semi-axis of (x) one can see that there exists a minimal stress state, and in the zone adjacent to the rear face of the wall, a maximal stress state. Therefore, along the positive semi-axis of x, as before:

$$\sigma = \frac{p(x)}{1+\sin\varphi} \quad , \quad \varphi = \frac{\pi}{2}$$

and along the rear face of the wall, from eq. $\sigma = \frac{p \sin \Delta}{\sin(\Delta - \kappa\,\delta)}$

with $\kappa = +1$, we take $q = \frac{\sigma \sin(\Omega - \omega)}{\sin \Omega}$, $\varphi = \beta + \frac{1}{2}(\Omega - \omega)$

where by definition : –

$$\sin \Omega = \frac{\sin \omega}{\sin \varphi} \qquad 0 \le \Omega \le \frac{\pi}{2}$$

ω : angle of internal friction on the horizontal plane $\le \varphi$

It will readily be seen that the problem of finding the pressure of a fill on a retaining wall is analogous to the problem of finding the restraining pressure on a foundation; these problems coincide when ($\beta = \pi$, $\omega = 0$).

B: Passive pressure of a fill on retaining walls

To find the passive pressure of the fill on a retaining wall assuming that the wall causes upward heave of the fill, so that ($\delta = -\omega \le 0$).

It is clear that in the zone adjacent to the positive semi-axis of x there will exist a maximal stress state, and in the zone adjacent to the rear face of the wall there will exist a minimal stress state.

Along the positive semi-axis of x which forms the boundary of the fill, one can have:

$$\sigma = \frac{p(x)}{1-\sin\varphi} \quad , \quad \varphi = 0$$

and along the rear face of the wall, from eq. $\sigma = \frac{p \sin \Delta}{\sin(\Delta - \kappa\delta)}$

with $\kappa = -1$, we must put : $q = \frac{\sigma \sin(\Omega + \omega)}{\sin\Omega}$, $\varphi = \beta - \frac{\pi}{2} + \frac{1}{2}(\Omega + \omega)$

It is easy to notice that the determination of the passive pressure of a fill on a retaining wall is analogous to the process of finding the failure pressure one foundation these problems are identical when ($\beta = \pi$, $\omega = 0$). The output and sketch of this case are shown in figure (8).

Discussion and Conclusion

- The discussion of the results depends basically on transition of failure load through each zone and the direction in which the failure stress will increase and the effect of the previous on the shape of each zone and final mesh.
    1- Zone one: It represents the active zone which lies under the surcharge along the α – lines failure load will

increase because of the effect of load which decreases with depth. So along the depth the failure load will increase, the matter that means the probability of failure becomes less. This also happened when the researcher transfers far from the original point (point of characteristic ordinates), be for the same reasons.

2- Zone two: It represents zone between active & passive zone (wave transition zone). The real shape for this zone is spiral; many procedures consider this zone as a sector of a circle in order to simplify the derivation. In this zone the transition of the failure load will be similar to zone one for the α – lines and the failure load will increase with the increasing of the distance from the applied load.
3- Zone three: Zone three represents the passive zone of the mesh, it is similar to zone (1) in the general shape. The transition of the failure load will continue in increasing with the distance of the (-) x and (+) y.

- ψ is measured from the x-coordinate for all zones it increases with the direction of flow of the mesh and decreases with the direction of α-line.
- 1^{st}, 2^{nd} & 3^{rd} boundary value problems were used as a technique for solving zone 1, 2 & 3 respectively. The conclusion there for will be:

  When one gets away from the loaded area, the effect of load will decrease gradually, so the failure load will increase. This is also applicable for the angle of inclination of failure load.
- The effect of inclination of both load & ground on the shape of all zones and final mish depends on the variation of angle of inclination & the variation of soil type by a simple observation to the results of the three examples, one can notice that:
- On (case one) when the load is vertical and the surface is horizontal (β = 0), it can be noticed that at the surface zone and on non-characteristic line the angle (ψ = 90) because at any point on the surface has a maximum stress of "p" equals to stress applied, while if there is any inclination in load on surface or both, ψ will be less than 90^{o}.
- Shape of zone (2) depends on three cases Case of loading, Unbalancing condition and Slop of ground.
- Generally by observing the shape of zone 2 in the three examples. When the inclination of load, ground or both, the size of the intermediate translation zone (2) will be smaller, and when

there is a case of conditions loading (unbalancing condition), it leads to a narrower area of zone 2, as detailed in the three solved problems.

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Symbols

σy normal stress in the y direction acting on element.
σx normal stress in the x direction acting on element.
τxy shear stress in the y direction acting on element.
p total principle stress, p′ effective stress (principle).
δ slope angle of (p) taking from the y direction.
$\acute{\delta}$ slope angle of (p′) taking from the y direction.
Φ angle of initial friction.
γ unit weight of the material.
ψ angle of the principle stress with the horizontal.
λ angle of the plane of equivalent stress.
ε angle of plane of failure, 2ε angle between (α ,β).
ω angle of internal friction of the horizontal plane.

Figure (1-a)

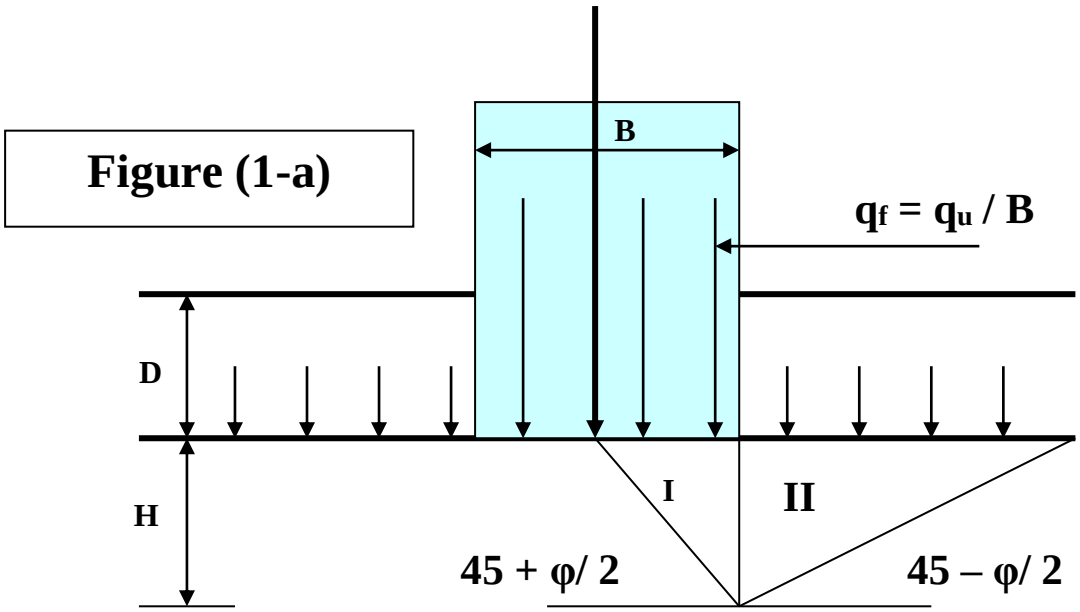

Figure (1-b)

$$H = \frac{B}{2}\tan(45+\varphi) = \frac{B}{2}\sqrt{K_P}$$

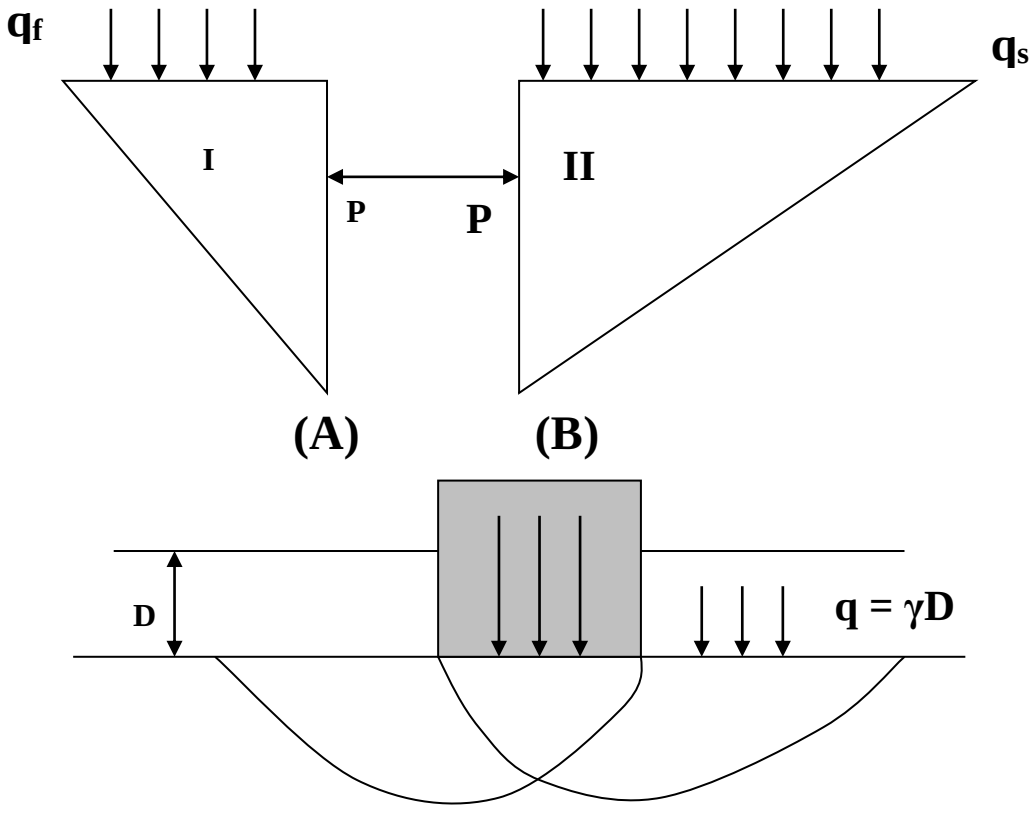

Figure (2):
3 zones failure mesh

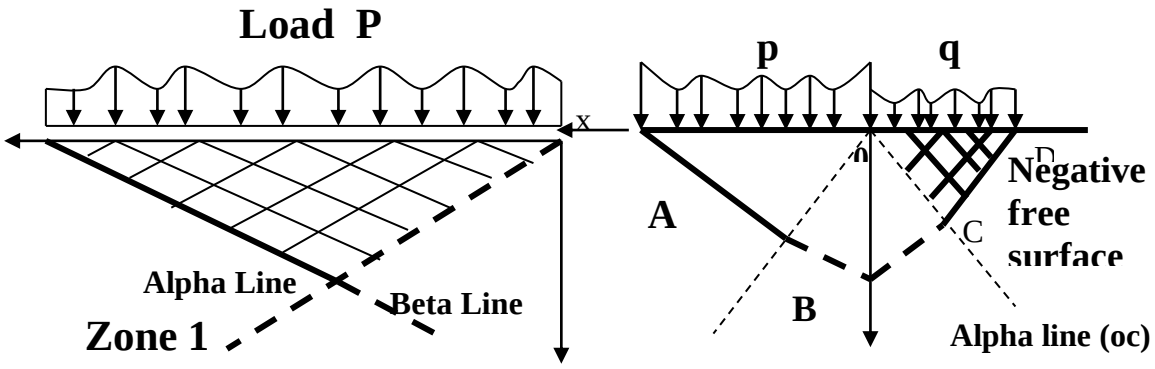

Figure (3)

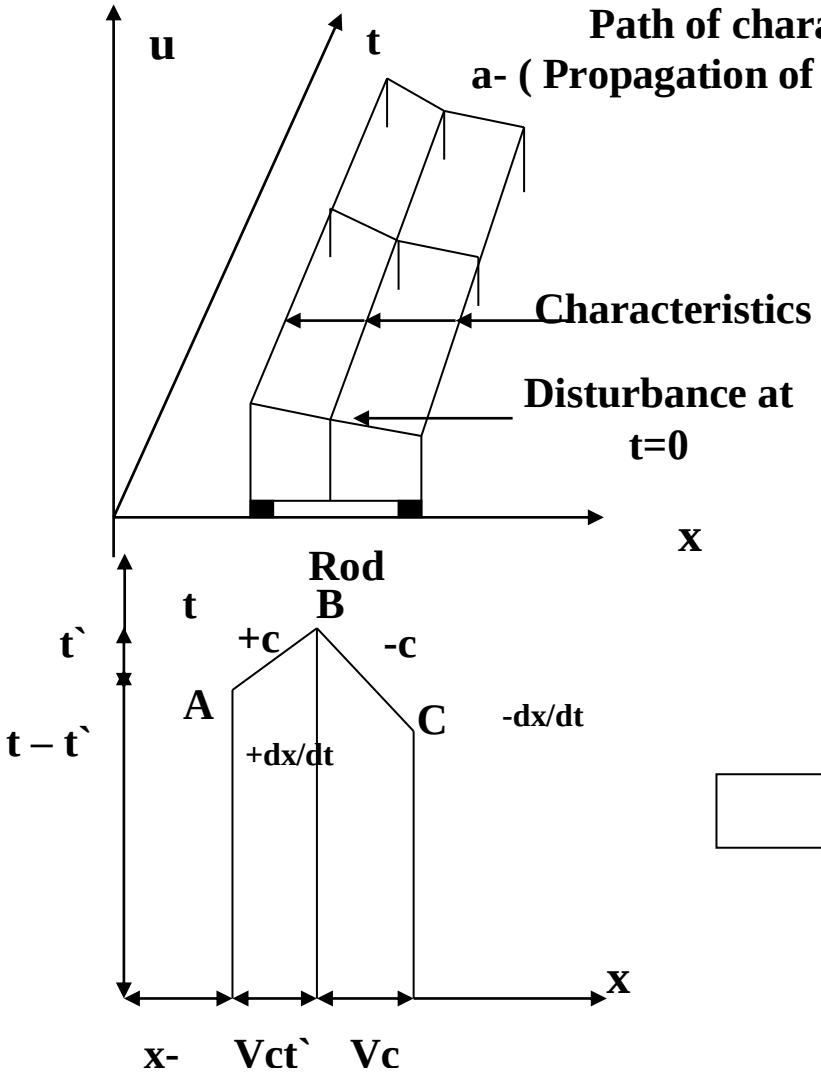

Figure (4)

b. Characteristics for linear wave

(Patterns of families of characteristics) or regions characteristics structure

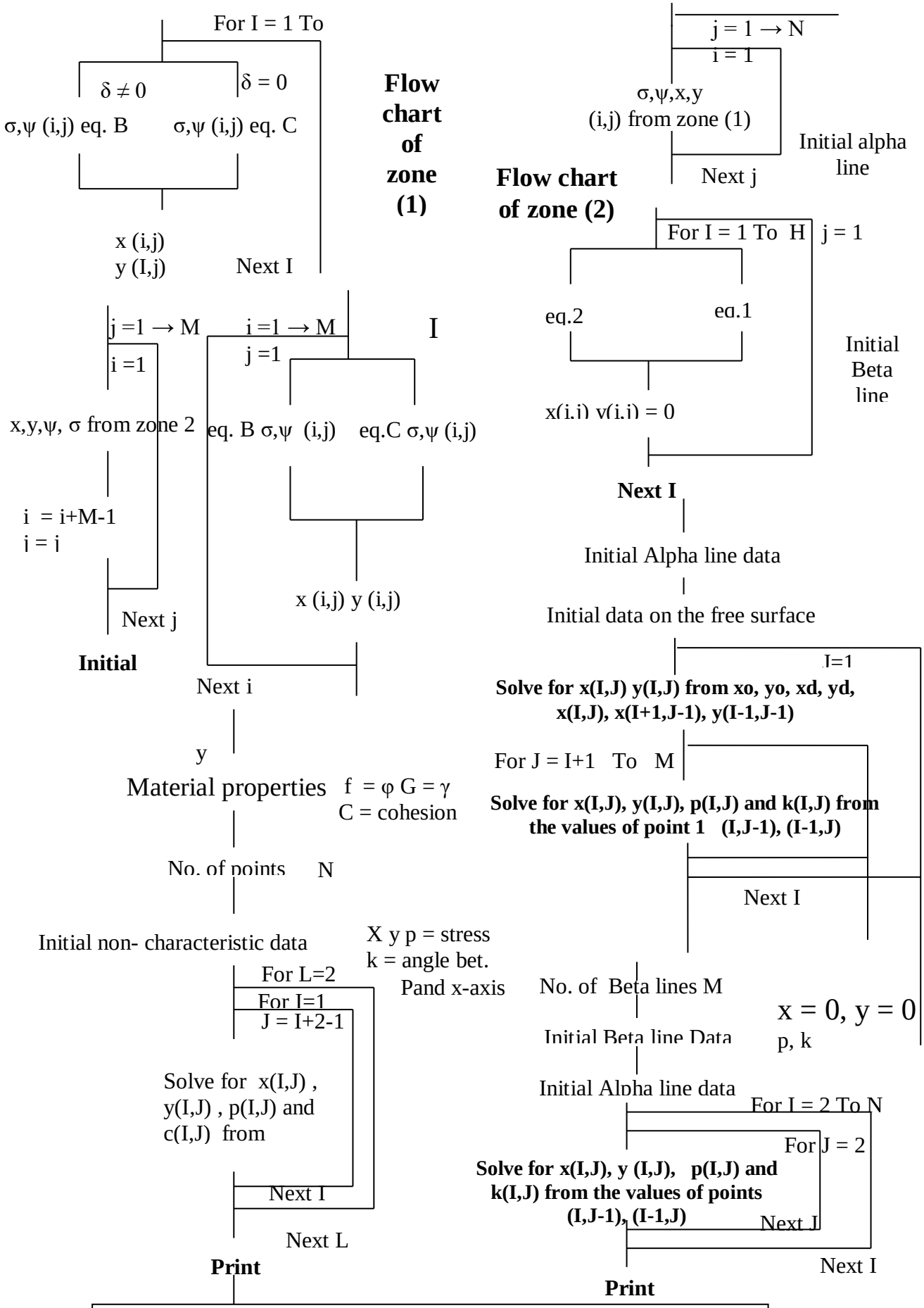

Figure (5): Flow chart of the program

MATERIAL PROPERTIES

INITIAL ALPHA LINES DATA

F= .5325 G= 1 C= 1

CAUCHY PROBLEM

No. OF POINTS=5

INITIAL NON CHARACTERISTIC DATA

I	J	X	Y	P	K
1	1	0.00	0.00	.67	1.57
2	2	.20	0.00	.80	1.57
3	3	.40	0.00	.93	1.57
4	4	.60	0.00	1.07	1.57
5	5	.80	0.00	1.20	1.57

SOLUTION OF ZONE (1)

I	J	X	Y	P	K
1	2	.09	.17	.86	1.63
2	3	.29	.17	.99	1.62
3	4	.49	.17	1.12	1.62
4	5	.69	.17	1.25	1.61
1	3	.17	.35	1.04	1.67
2	4	.38	.35	1.17	1.66
3	5	.58	.35	1.30	1.65
1	4	.25	.52	1.22	1.70
2	5	.45	.52	1.35	1.69
1	5	.32	.69	1.40	1.72

MIXED PROBLEM

INITIAL ALPHA LINES DATA

I	J	X	Y	P	K
1	1	0.00	0.00	.35	2.12
1	2	.02	.25	.60	1.98
1	3	.05	.48	.81	1.94
1	4	.08	.69	1.01	1.92
1	5	.12	.90	1.20	1.91

P,K,VALUES ON THE FREE SURFACE

I	J	P	K
1	1	.35	2.12
2	2	.52	2.12
3	3	.68	2.12
4	4	.84	2.12
5	5	1.00	2.12

X0=0;Y0=0;Xd=10;Yd=10

SOLUTION OF ZONE (3)

I	J	X	Y	P	K
2	3	-.12	.60	.76	2.03
2	4	-.10	.83	.97	1.99
2	5	.07	1.05	1.16	1.97
3	4	-.24	.93	.91	2.10
3	5	-.23	1.16	1.12	2.02
4	5	-.36	1.25	1.07	2.07

GOURSAT PROBLEM

No. OF BETA LINES IN ZONE (2)=5

INITIAL BETA LINES DATA

I	J	X	Y	P	K
1	1	0.00	0.00	.67	1.57
2	1	0.00	0.00	.57	1.71
3	1	0.00	0.00	.48	1.58
4	1	0.00	0.00	.41	1.98
5	1	0.00	0.00	.35	2.12

INITIAL ALPHA LINES DATA

I	J	X	Y	P	K
1	1	0.00	0.00	.67	1.57
1	2	.09	.17	.86	1.63
1	3	.17	.35	1.04	1.67
1	4	.25	.52	1.22	1.71
1	5	.32	.69	1.40	1.72

SOLUTION OF ZONE (2)

I	J	X	Y	P	K
2	2	.08	.20	.77	1.74
2	3	1.14	.38	.96	1.76
2	4	.21	.57	1.14	1.78
2	5	.27	.76	1.32	1.79
3	2	.06	.22	.70	1.83
3	3	.11	.42	.89	1.83
3	4	.16	.62	1.08	1.84
3	5	.21	.81	1.27	1.84
4	2	.04	.24	.46	1.91
4	3	.08	.45	.85	1.90
4	4	.12	.66	1.04	1.89
4	5	.16	.86	1.23	1.88
5	2	.02	.25	.60	1.98
5	3	.05	.48	.81	1.94
5	4	.08	.69	1.01	1.92
5	5	.12	.90	1.20	1.91

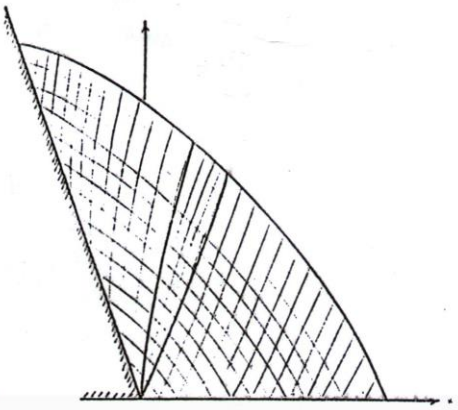

Figure (6)

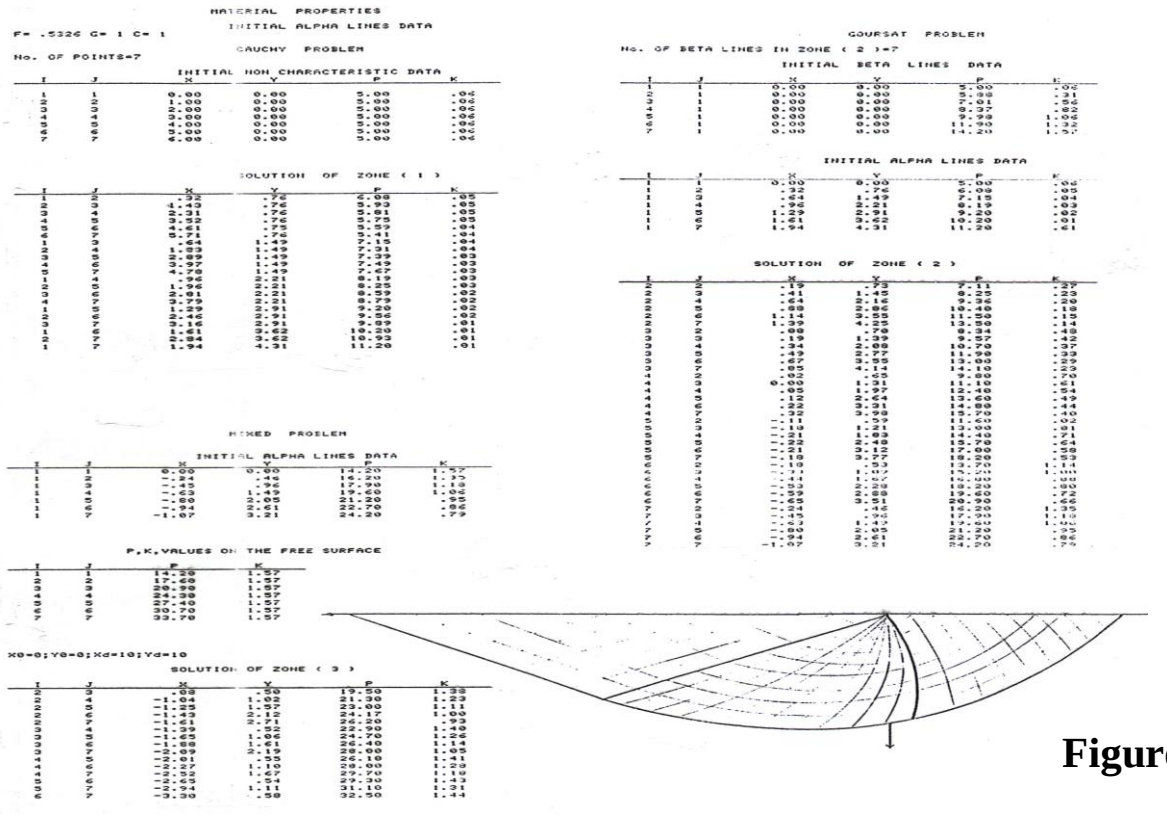

Figure (7)

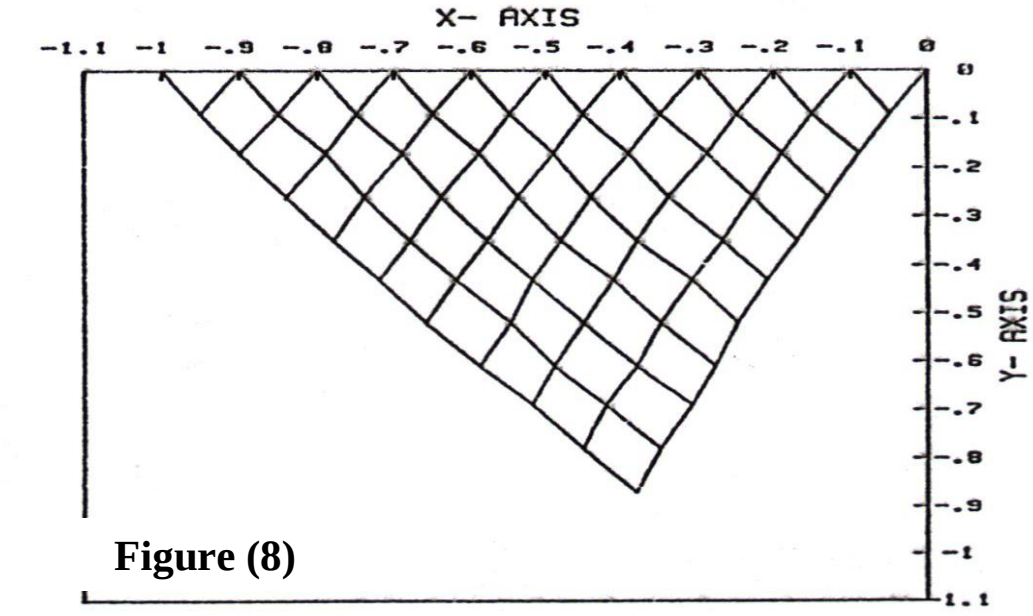

Figure (8)

);ALPHA-BETA LINES FOR LIMIT EQUILIBRIUM OF FOUNDATION BY THE 1ST. BOUNDARY VALUE PROBLEM

FINITE DIFFERENCE PROGRAM FOR SOIL PROPERTIES BY THE METHODE OF CHARACTERISTICS

H.Y.Hameed (M.Sc.)

& Dr. K.F.Sideek

ABSTRACT

This work deals with the limit equilibrium technique, which is more approximate than the limit analysis. Besides it deals with the mechanism of falling surfaces and slip surfaces occurring in soils of different cases. The procedure followed in this work is to put the equilibrium equations by taking element from soil under general condition. Solving these two partial differential equations by the method of characteristics in order to get two hyperbolic ordinary equations, then solving these equations by the finite difference method using computer algorithm programmed using *FORTRAN 77* language in order to give the complete solution of the problem in concern. The proposed solution will provide the advantages to deal with the conditions of soil at any point (stresses, failure slopes) under different effects such as surcharge, water effect, retaining walls effect... etc. This means any failure surface of any type of distribution of load or stress in any kind of soil can be solved using the proposed technique. Finally many checks are made to insure the validity of the proposed technique.

Introduction and theoretical background: The soil must be capable of carrying the loads from any type of structure founded upon it with acceptance settlements but without causing any type of failure.

So, it is necessary to investigate phenomenon, stress and deflection (base shear resistance and settlement) for any structure. In many cases settlement criteria will control the allowable bearing capacity; however, there are also a number of cases where base shear dictates the recommended bearing capacity, e.g. the allowable bearing capacity for footings on unsaturated cohesive soil which is often based on the unconfined compression strength, which is turn a simplified version of the bearing capacity equations. Structure founded on soft soils such as liquid storage tanks and mats may be more susceptible to a base